

# A graphical view of distance between rankings: the Point and Area measures

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# Outline

- Classification of rank similarity measures
- Spearman foot-rule and Kendall distance
- Point and Area measures
- Measures of effectiveness
- Conclusions

# Comparing ranked list

- Search engines effectiveness can be measured by analyzing their visible outcomes
  - lists of documents ranked in descending order of relevance to a given topic

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Abstracts due date, October 10, 2014 November 10, 2014. Pre-registration / Hotel bookings, January 2015. Manuscript submission due date, February 20, 2015.

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Event Description. ICR IIR 2015, 24th IIR International Congress of Refrigeration is going to be scheduled from 16 August, 2015 to 22 August, 2015.

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Latest news . Si è tenuta il 28 aprile 2015 a Milano la 2ª edizione di Italy Insurance Forum

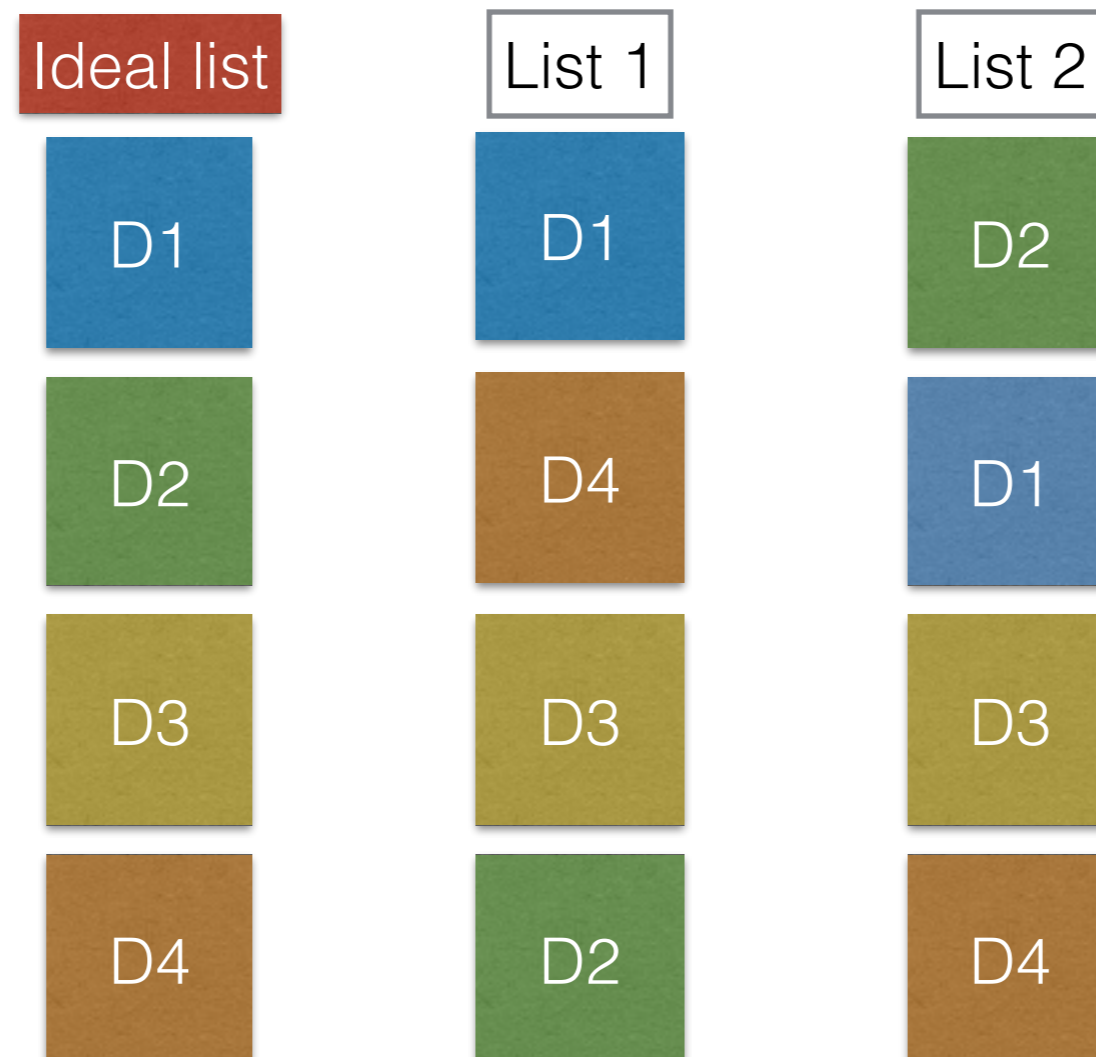
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General Chairs . Domenico Dato (Tiscali) Raffaele Perego (ISTI-CNR) Program Chairs . Paolo Boldi (Università di Milano) Fabrizio Sebastiani (Qatar Computing Research ...

# Comparing ranked list

- Correlation among rankings can be used to assess the search engines effectiveness



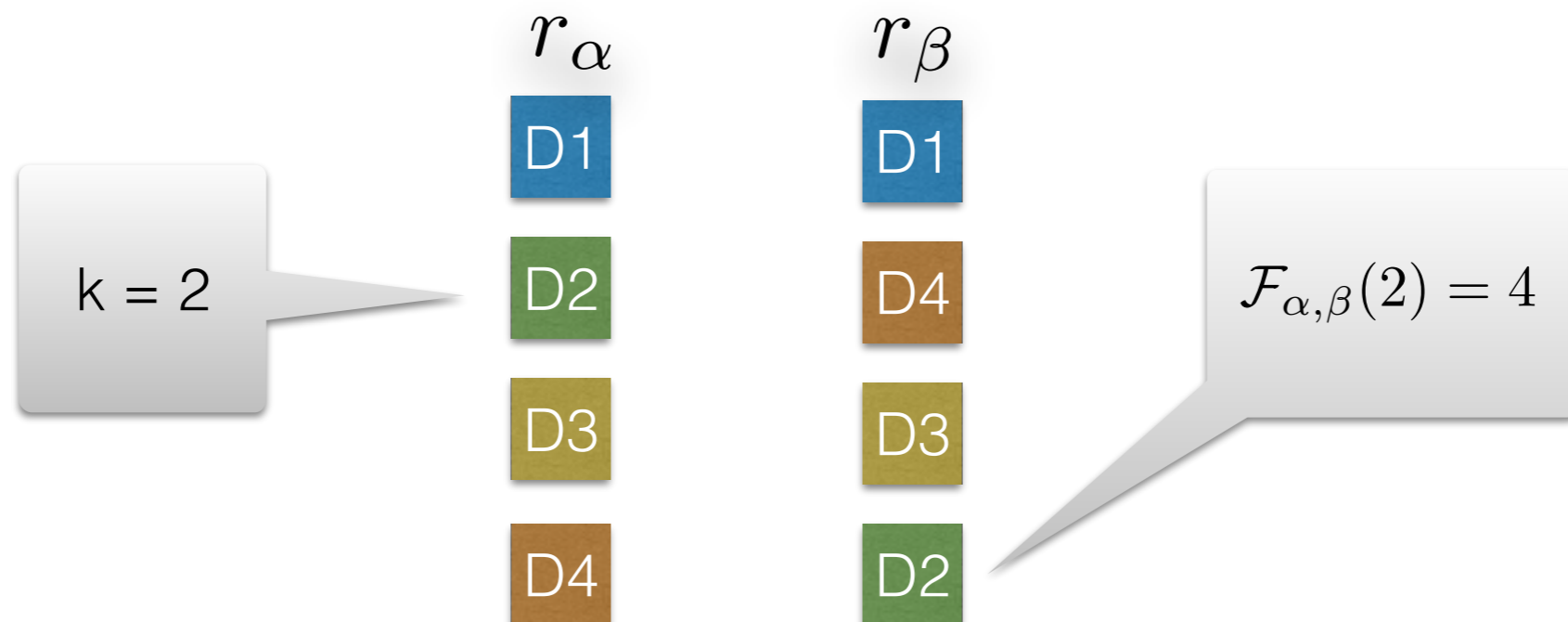
# Classification of rank similarity measures

- Weighted / non-weighted
  - Exchanges in the ordering at the top of the ranking are more significant than those at the bottom
  - Any perturbation has the same importance
- Conjoint / non-conjoint
  - Two rankings have the same elements
  - Some elements in one list do not appear in the other

# Find elements in a ranked list

- Map index of a document from one list to the other
- Given  $k$ -th element in list  $r_\alpha$ , return index of that element in list  $r_\beta$

$$\mathcal{F}_{\alpha,\beta}(k) = \text{idx}_\beta(r_\alpha(k))$$



# Spearman foot-rule

- Compute the total element-wise misplacements between two ranked lists
- Non-weighted, conjoint

$$S_{\alpha,\beta}(i) = \sum_{k=1}^i |\mathcal{F}_{\alpha,\beta}(k) - k|$$

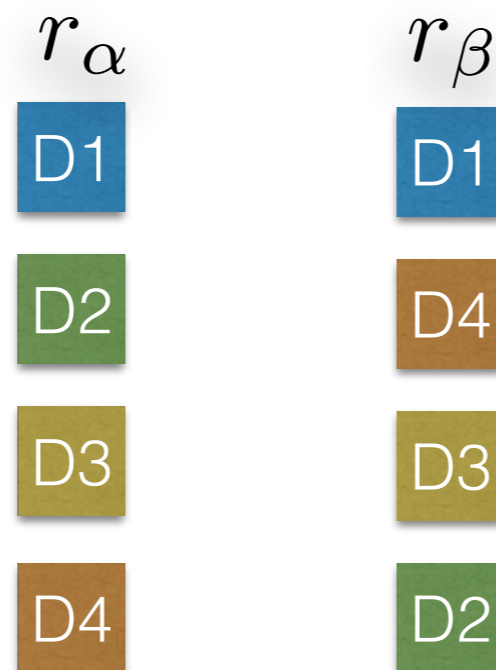


$$S_{\alpha,\beta}(4) = 0 + 2 + 0 + 2$$

# Kendall distance

- Number of adjacent swaps that are necessary to reorder one list as the other
- Non-weighted, conjoint

$$K_{\alpha,\beta} = \sum_{(i,j):i<j} \tilde{K}_{i,j}(r_{\alpha}, r_{\beta})$$



$$K_{\alpha,\beta} = 0 + 2 + 1$$



# Spearman and Kendall

$$S_{\alpha,\beta}(i) = \sum_{k=1}^i |\mathcal{F}_{\alpha,\beta}(k) - k|$$

$$K_{\alpha,\beta}(i) = \sum_{k=1}^i (\mathcal{F}_{\alpha,\beta}(k) - k) + \underbrace{(r_{\alpha}[1 : k] \cap r_{\beta}[(\mathcal{F}_{\alpha,\beta}(k) + 1) : n])}_X$$

# Point-wise distance

- Spearman without absolute value (!)
- Non-weighted, conjoint

$$P_{\alpha,\beta}(i) = \sum_{k=1}^i (\mathcal{F}_{\alpha,\beta}(k) - k)$$



$$P_{\alpha,\beta}(4) = 0 + 2 + 0 - 2$$

# Spearman, Kendall, Point-wise

$$S_{\alpha,\beta}(i) = \sum_{k=1}^i |\mathcal{F}_{\alpha,\beta}(k) - k|$$

$$K_{\alpha,\beta}(i) = \sum_{k=1}^i (\mathcal{F}_{\alpha,\beta}(k) - k) + \underbrace{(r_{\alpha}[1 : k] \cap r_{\beta}[(\mathcal{F}_{\alpha,\beta}(k) + 1) : n])}_X$$

$$P_{\alpha,\beta}(i) = \sum_{k=1}^i (\mathcal{F}_{\alpha,\beta}(k) - k)$$

# Visualization analysis

## Point and Area measures

Compute until rank i

1 10

1 2 3 4 5 6 7 8 9 10

New ranking list

Area-wise

Worst ranking

Ideal list r\_a

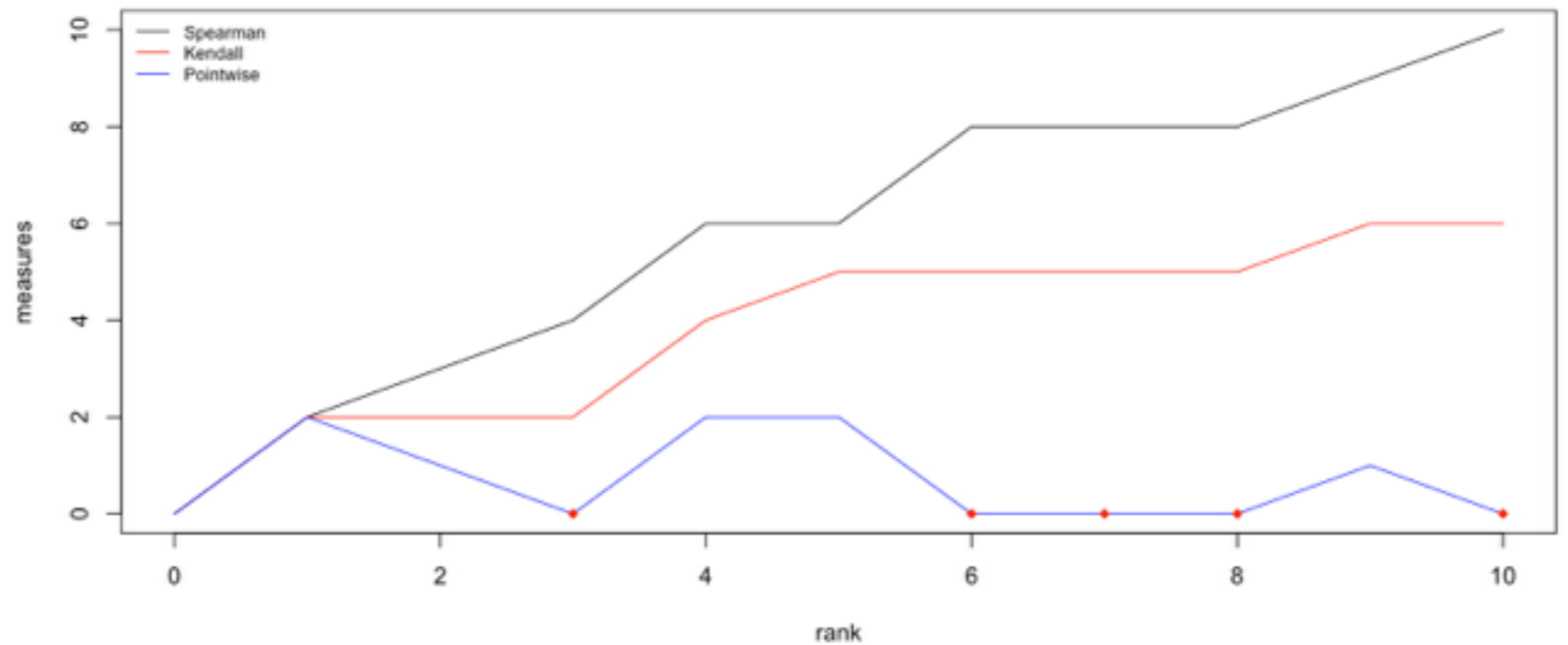
1 2 3 4 5 6 7 8 9 10

Ranking list r\_b

2 3 1 6 5 4 7 8 10 9

Example

rand  ex1  ex2  ex3  ex4



spearman	kendall	pointwise	areawise
10.00	6.00	0.00	8.00

spearman.rho	kendall.tau	area.corr
0.01	0.02	0.95

# Visualization analysis

## Point and Area measures

Compute until rank i

1 6 10

1 2 3 4 5 6 7 8 9 10

New ranking list

Area-wise

Worst ranking

Ideal list r\_a

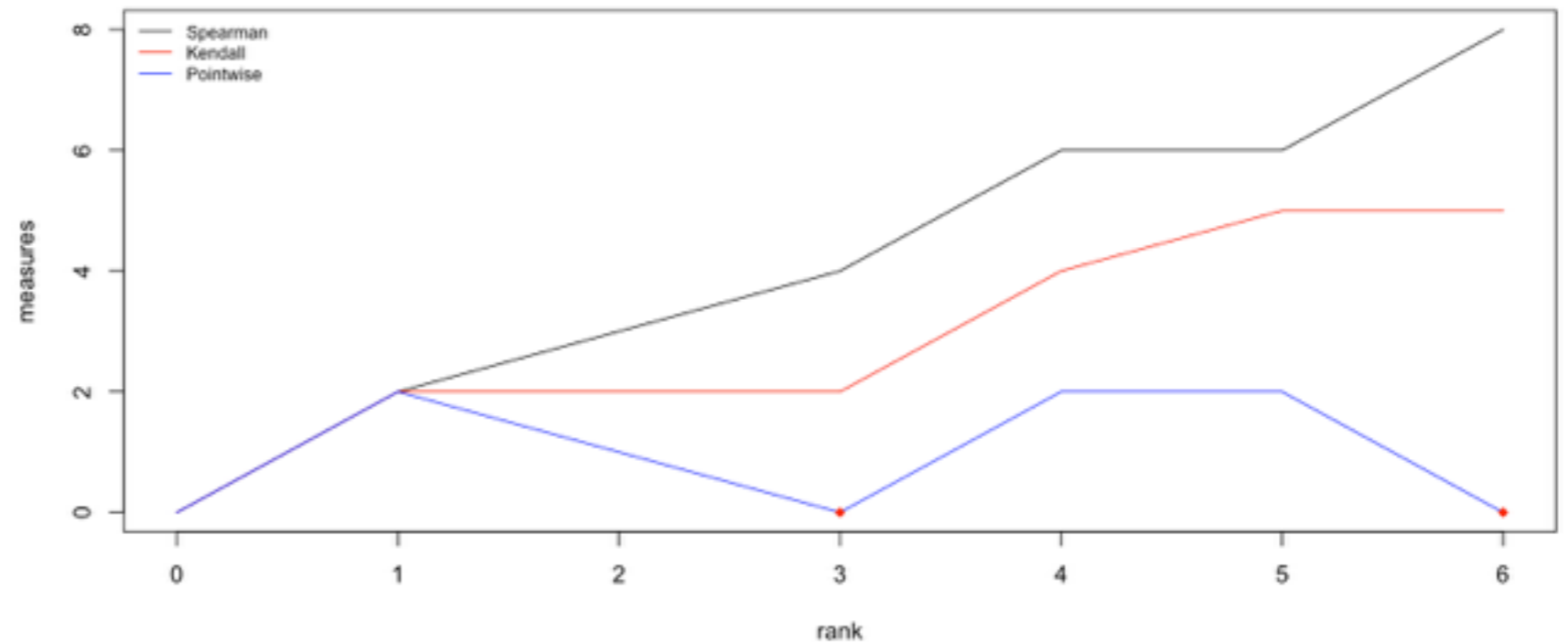
1 2 3 4 5 6

Ranking list r\_b

2 3 1 6 5 4 7 8 10 9

Example

rand  ex1  ex2  ex3  ex4



spearman	kendall	pointwise	areawise
8.00	5.00	0.00	7.00

spearman.rho	kendall.tau	area.corr
0.37	0.33	0.93

# Area-wise distance

- The area-wise measure considers the area formed by the segments between two adjacent points (point distance) and the x-axis.
- $h$  is the height of each trapezoid. It can be tuned to weight misplacements that occur in different part of the ranking list.

$$A_{\alpha,\beta}(i) = \sum_{k=1}^i \frac{(P_{\alpha,\beta}(k-1) + P_{\alpha,\beta}(k))}{2} h$$

# Area-wise normalization

- Divide the area of a relevance list at rank  $i$  by the largest obtainable area given by the worst possible ranking.
- It is in the  $[0, 1]$  range, where 0 indicates the ideal case and 1 the worst case

$$nA_{\alpha, \beta} = \frac{A_{\alpha, \beta}}{A_{\alpha, \beta}^*}$$

# Area Correlation (A-corr)

- A-corr is an indicator of the correlation between two ranked lists
- It is in the range  $[0,1]$ , where 0 indicates that two lists are not correlated and 1 that they are the same

$$\text{A-corr}_{\alpha,\beta} = 1 - nA_{\alpha,\beta}$$

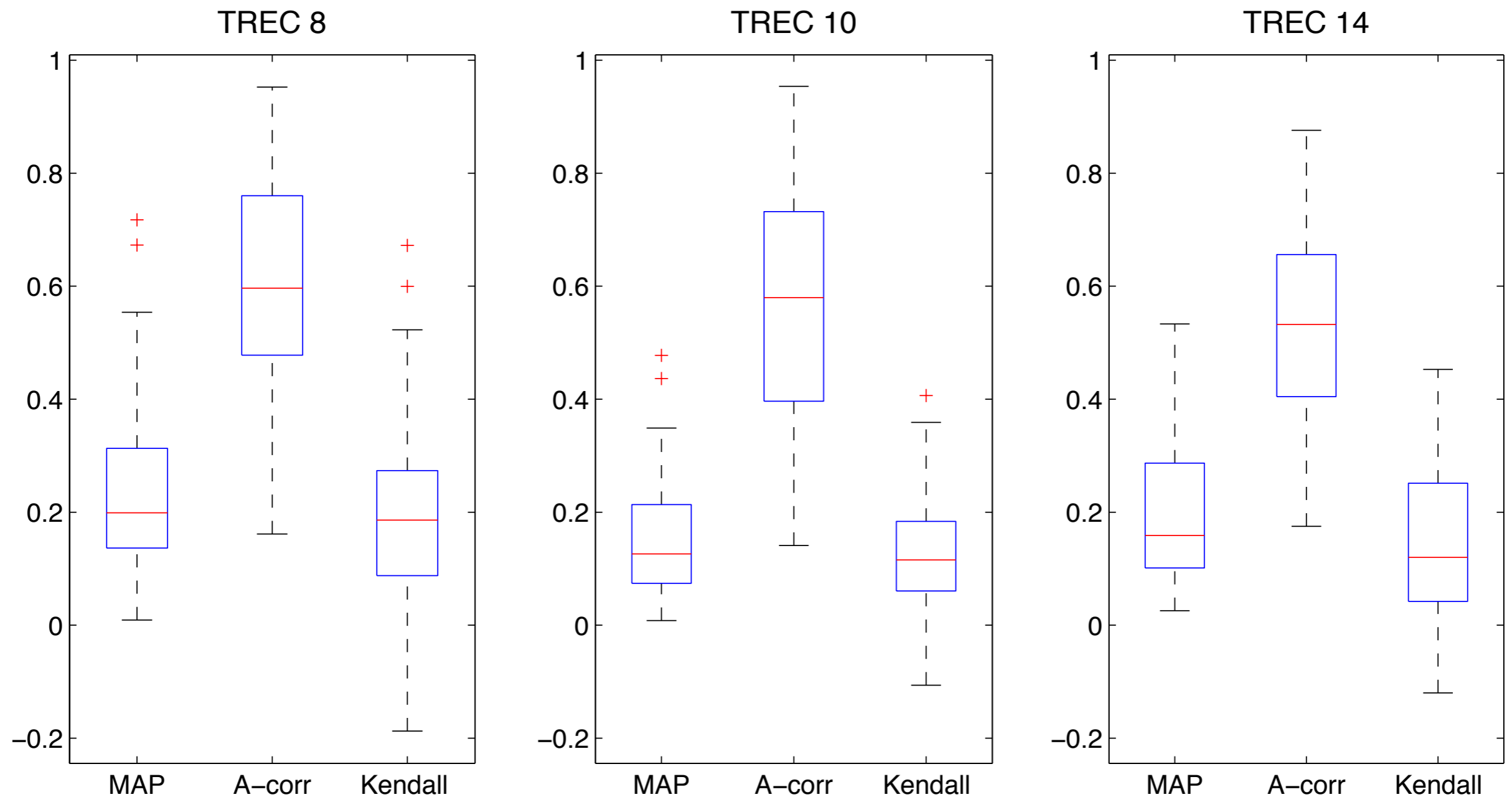


# A-corr as an effectiveness measure (quantitative)

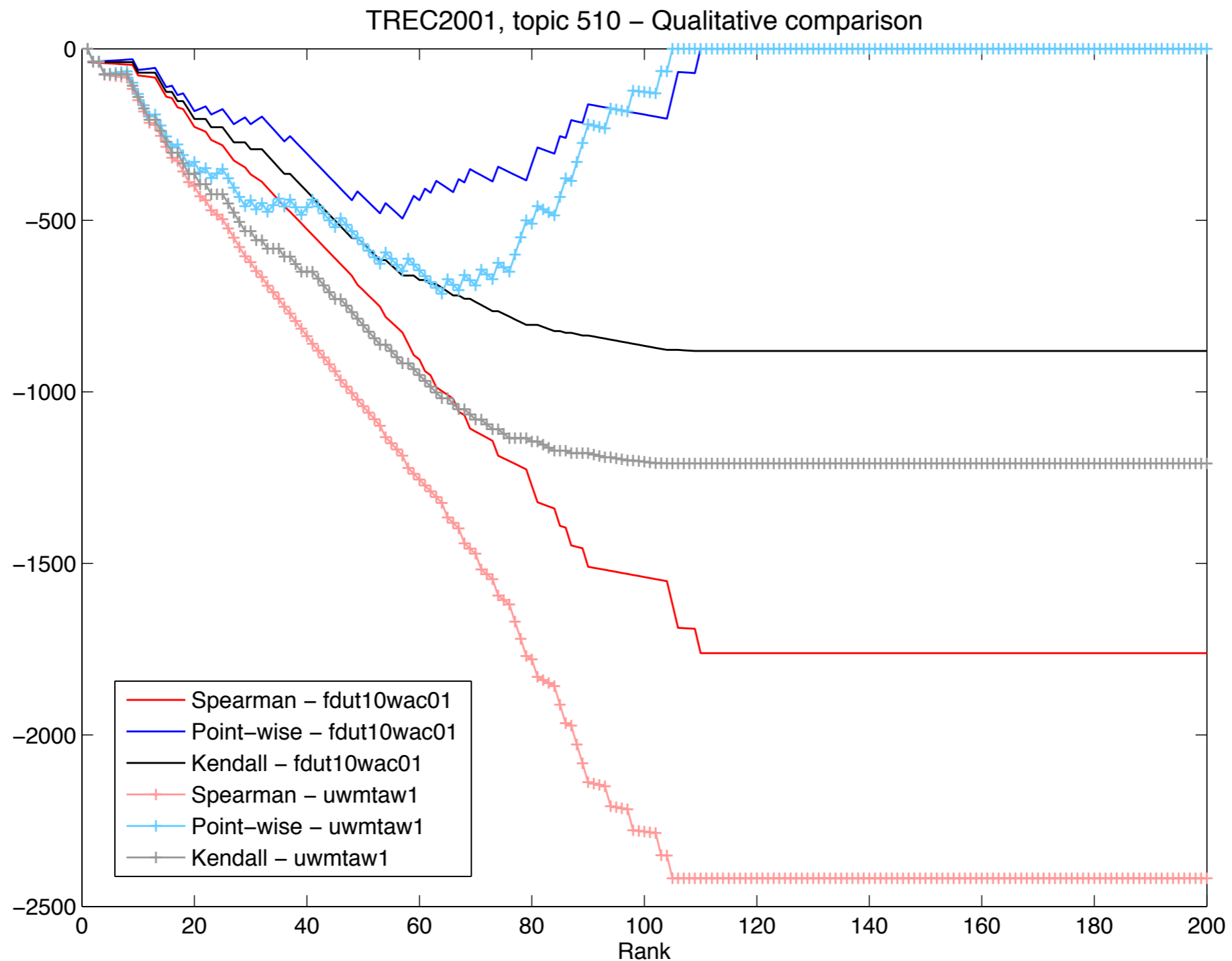
- We can calculate the point-wise measure by considering the relevance of documents



# A-corr on TREC test collections



# Point-wise as an effectiveness measure (qualitative)



# Conclusions

- A correlation and an effectiveness measure for qualitative and quantitative evaluation
- We plan to:
  - compare A-corr with the Twist measure (Cumulative Relative Position)\*
  - analyze its stability, sensitivity and correlation with other measures
  - define a weighted measure to model user behavior

\* N. Ferro, G. Silvello, H. Keskustalo, A. Pirkola and K. Järvelin (2015), The Twist Measure for IR Evaluation: Taking User's Effort into Account *Journal of the Association for Information Science and Technology (JASIST)* in print (<http://onlinelibrary.wiley.com/doi/10.1002/asi.23416>).